Parabolic Prediction of the Timing of Specified Peaks for World Conventional Oil Production

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Abstract

The parabolic technique previously devised to project geological assessments of undiscovered resources of conventional oil was modified to determine the date of occurrence of assumed values of the peak in world conventional oil production. The objective was to check the self-consistency of a range of typical values for the peak with the timing often quoted in the energy literature. Four cases for the unconstrained peak were chosen and the results were compared with those from a Reference Case that appeared in an earlier paper which was based upon the 1990 World Resources Assessment of the U.S. Geological Survey. The quantity of oil expected to be produced after 2003 was also calculated for each case.

The four cases chosen were: A - 85 million barrels per day (31.03 gigabarrels per year) with the peak found to occur in 2022.2; B - 90 mbbls/day (32.85 GB/Y) with the peak in 2031.8; C - 95 mbbls/day (34.68 GB/Y) with the peak in 2041.3; and D - 100 mbbls/day (36.50 GB/Y) with the peak in 2050.6. The timing of the peak was found to vary linearly with its value in the parabolic model. The quantity of oil expected to be produced after 2003 was also found to be approximately linear with the date of the peak. These values were so high that it is unlikely that the peak in the world production of conventional oil will exceed 90 million barrels per day. The Reference Case calculated from the resource assessment in the earlier note predicts the peak to be in 2015.4 which seems more probable.

Introduction

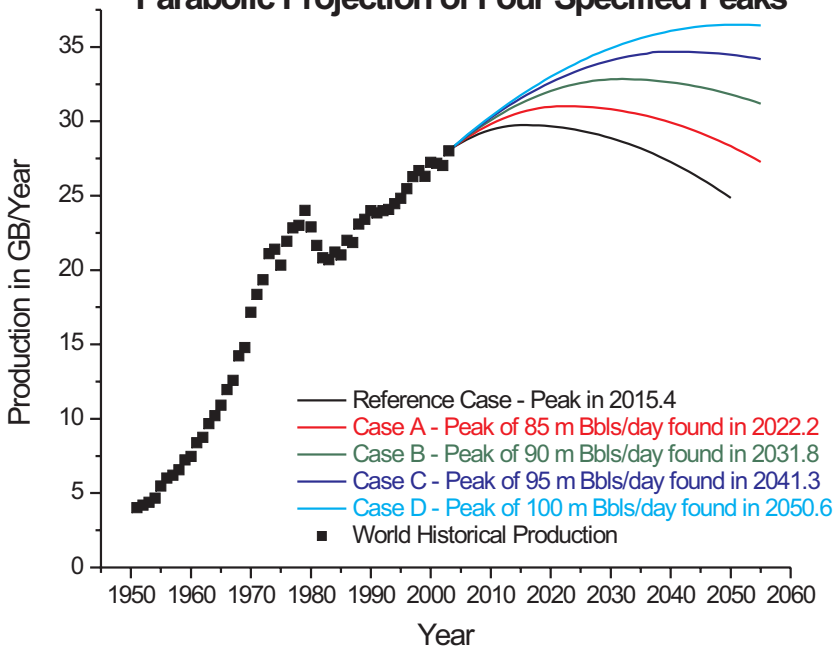
This note is one of a series of explorations of the properties of the parabolic modelling approach to world conventional oil production reviewed in Reference 1. The object here is to determine the date of the peak in production of conventional oil of the world given an exogenously specified maximum output. The parabolic technique is applied to examine the self-consistency between the values frequently quoted for specified ranges of peak production in relation to the corresponding expected timing. The calculation employs two historical values for world production taken from the *BP Statistical Review of World Energy*[2]. The two years selected were 2003, the most recent year for which statistics were available from this source at the time of writing, and 1993, which served as the staging point for the parabola. This latter year was chosen because it allowed for the consumption of a significant quantity of oil (264 GB) in the intervening ten-year period to add accuracy to the calculation but was recent enough to avoid the period of severe dislocation in production that occurred during the events of the 1970s and their immediate aftermath. With the selection of these two years, no other data was required to prepare the parabolic projection for each specified peak. Different parabolas will pass through the graphical points for these two years for each value of the peak specified. The timing of the peak is obtained from the properties of the projection as described below.

Methodology

The following steps were employed in the calculation:

1. The basic equation for production versus time derived in Reference 1 was modified by substituting 2/3P for Q/T which was taken from the parabolic area formula of Q = 2/3 PT, and with the definition

Figure 1
Parabolic Projection of Four Specified Peaks



of r = t/T. The basic production equation thus becomes:

$$p_a = 6Q/T^2\ t_a(1-t_a/T) = 6Q/T\ r_a(1-r_a) = 4P\ r_a(1-r_a)$$

where p_a = production in gigabarrels (GB) for the year a, Q = area of the staged parabola, $r_a = t_a/T$ where t_a is the time in years from the zero origin of the parabola to the year a, and T is the returning intercept of the parabola on the time or X axis. All values of t_a lie between 0 and T.

2. The value of r is then determined for 2003 and 1993 (ten years apart) from the actual historical production for these two years and the specified value for the peak production, P, by solving the two resulting quadratic equations below for each case. In this note, the values of r for 2003 and 1993 were designated r_2 and r_1 with the corresponding known historical production for these years p_2 and p_1 respectively.

$$r_2^2 - r_2 + p_2/4P = 0 \text{ and } r_1^2 - r_1 + p_1/4P = 0$$

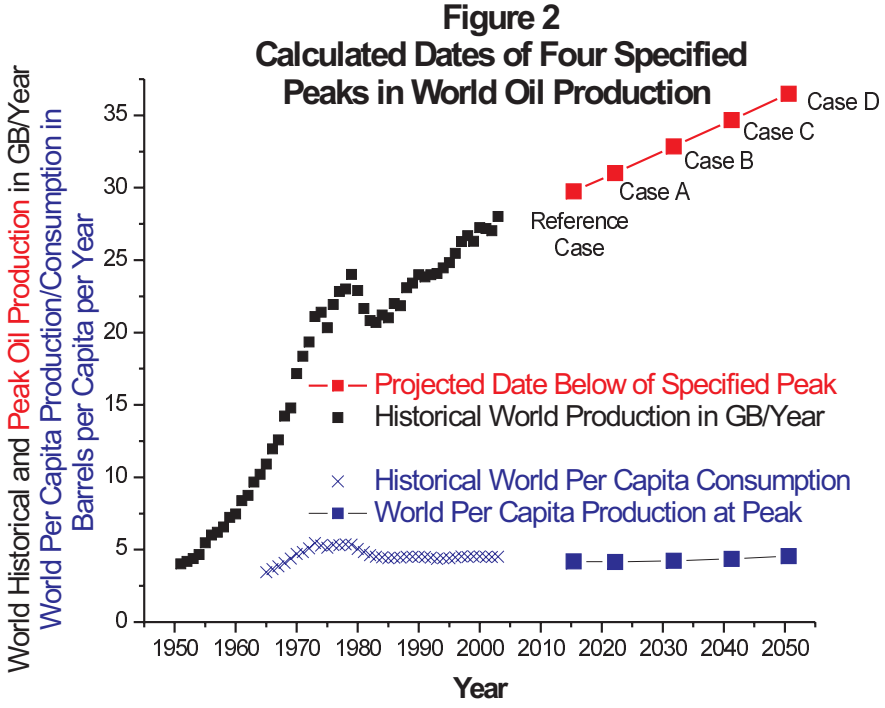
3. The values derived for r_2 and r_1 taken from the lower of the two solutions for each of the two quadratic equations were then subtracted to determine the value of T as follows:

$$r_2 - r_1 = (t_2 - t_1)/T = 10/T \text{ thus } T = 10/(r_2 - r_1).$$

Having determined the value for T, the value of Q can then be obtained from 2/3PT. From a knowledge of T, t_1 and t_2 may also be evaluated from r_1 and r_2 for each of the four cases of the values specified for the peak, P. This provides sufficient information to permit a parabola to be drawn that will pass through the production for the two years selected (2003 and 1993) and the particular value selected for P for each case by using the following equation to prepare the graph of production versus time:

$$p = 4P\ (t/T)(1 - t/T)$$

4. The year of the zero point of the parabola in terms of years is determined by deducting t_2 from 2003 or t_1 from 1993 which should give the same result. The year of the timing of the peak P is found when T/2 years are added to the date of the zero point.



5. Four values were selected for the unconstrained peak: Case A - 85 million barrels per day (31.025 GB/Year); Case B – 90 mbbl/d (32.850 GB/Y); Case C – 95 mbbl/d (34.675 GB/Y); and Case D – 100 mbbl/d (36.500 GB/Y). Though arbitrary, the cases selected for P are in the range of frequently quoted values for the peak in the world production of conventional oil.

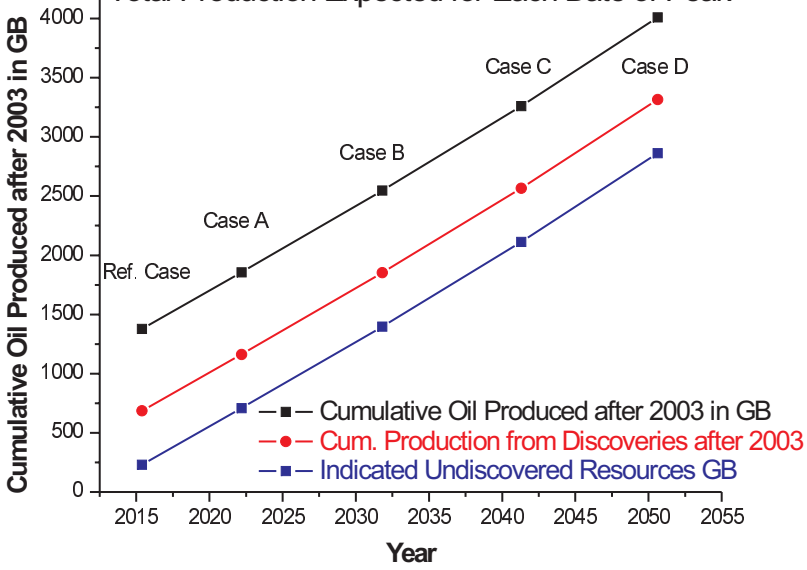
6. The cumulative production between 1993 and 2003 calculated from the individual parabolas drawn for Cases A, B, C, and D was checked against the actual historical cumulative production over this period of 263.6 GB. These values were found to agree within 1% as listed in Table 1. This agreement was considered satisfactory though the error was found to increase systematically with higher selected values of the peak production, P.

7. The production by year after 2003 was projected for each of the four cases as illustrated in Figure 1. The Reserves Addition of 672 GB taken from the Year 2000 Resource Assessment of the U.S. Geological Survey was included only after the peak has passed as was the practice in Reference 3 and thus this procedure does not affect the calculation for the determination of the timing of the peak itself.

8. The Reference Case was taken from the previous note[3]. In it the world Resource Assessment of the U.S. Geological Survey was projected to the prediction of a peak of 81.51 mbbl/d (29.751 GB/Y) in 2015.4 which is to be compared with the actual production of 76.78 mbbl/d (28.02 GB/Y) in 2003 reported in the *BP Statistics*.

An estimate of the total oil to be produced after 2003 for each of the four cases appears in Table 1. This value was calculated by deducting the cumulative value up to 2003 from the area of each parabola (not the cumulative world production to this year). This quantity does not include the Reserves Addition to preserve the distinction between oil produced from new discoveries and that due to the Reserves Addition made in Reference 3 in that two quite different processes – exploration versus reservoir gains - are involved. The Reference Case also included an estimate of the expected production only from that oil discovered before 2003 of 693.4 GB. This latter quantity was deducted from the total oil to be pro-

Figure 3
World Cumulative Oil Production after 2003
Total Production Expected for Each Date of Peak



duced after 2003 for each of the cases to estimate the quantity of oil to be produced from oil as yet undiscovered by 2003. This value is also reported in Table 1 below.

Table 1

	Refer. Case	Case A	Case B	Case C	Case D
Assumed Peak Production, P million bbls/day	81.50	85	90	95	100
Assumed Peak Production, P GB/Year	29.75	31.03	32.85	34.68	36.50
Date Calculated for Peak Production	2015.4	2022.2	2031.8	2041.3	2050.6
Total Production after 2003 GB	1379	1856	2546	3260	4010
Indicated Undiscovered Resources GB	231	708	1398	2112	2862
Prod. from Oil Discovered after 2003 GB	685	1163	1853	2567	3316
Per Capita Production at Peak bbl/person	4.19	4.16	4.23	4.36	4.56
Error in Cum.Production 2003 to 1993	0.46 %	0.65 %	0.79 %	0.87 %	0.92 %

Results

The four parabolas for Cases A, B, C, and D appear in Figure 1 together with the Reference Case taken from the previous note. A plot of historical world oil production is also included in this graph. The dates of the resulting peaks for the four Cases appear in Figure 2 along with that previously calculated for the Reference Case. It may be seen that the relationship between the peaks and the dates when they are projected to occur is linear. For this reason, intermediate values may be interpolated and perhaps extrapolated with confidence. Though not the subject of this paper, this curve may also be used in reverse: the expected peak production may be read off the graph for a specified date which determines the quantity of oil consistent with the parabolic relationship.

The world per capita production at the time of these peaks is also plotted in Figure 2. This calculation is based upon a world population scenario characterized by a conservative peak of eight billion people in 2050. These values are essentially constant but may be increasing slightly over time as listed in Table 1. Historical world per capita oil consumption is also plotted for comparison which for this purpose may be taken as essentially equal to world per capita production. This ratio has been effectively constant for the last twenty years and is well past its peak. This value should not increase very much if at all with time.

Two nearly straight lines appear in Figure 3 that relate (1) the cumulative production of oil by the world after 2003 and (2) the cumulative production from oil discovered after 2003 with each date specified for the peak.

The world published reserves at the end of 2003 of 1148 GB[2] were deducted from the Total Production after 2003 Table 1 to estimate the Indicated Undiscovered Resources in Table 1. The large quantities predicted by the parabolic modelling technique for cumulative post 2003 production and the Indicated Undiscovered Resources makes it unlikely that the peak will exceed 90 million barrels per day (32.85 GB/Y) of conventional oil. This value is lower than most economic projections.

References

1. J.H. Walsh, *Procedure for the Parabolic Projection of Geological Assessments of Conventional Oil and Gas Resources with Examples*, Revised January 2004. (Web: pages.ca.inter.net/~jhwalsh/wpara1.html)

2. *BP Statistical Review of World Energy*, June 2004. (Web: www.BP.com)

3. J.H. Walsh, *Note on Testing the Adequacy of Discovery Rates of Conventional Oil to Avoid the Formation of a Plateau Peak in World Production*, August 2004. (Web: pages.ca.inter.net/~jhwalsh/wcumdiff.html)

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