Procedure for the Parabolic Projection of Geological Assessments of Conventional Oil and Gas Resources with Examples

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Abstract

There is now major concern about the extent of the world's endowment of conventional oil resources and the North American supply of natural gas. This paper presents the details of the parabolic projection technique for the interpretation of geological assessments of undiscovered resources of conventional oil and gas in terms of production-time curves . The object is to present the methodology that was devised over the last decade in sufficient detail to allow this type of projection to be used and interpreted readily by others. Specific examples of its application are taken from some recent papers.

The examples include a projection of the Mean Value for the world's conventional oil resources published by the U.S. Geological Survey in 2000, the techniques devised for dealing with the Reserves Addition and the problem posed by the existence of idle capacity in the global oil system, and the 'plateau peak' now likely in Canadian natural gas production. The procedure used to calculate these special cases is presented along with the results in raphical form.

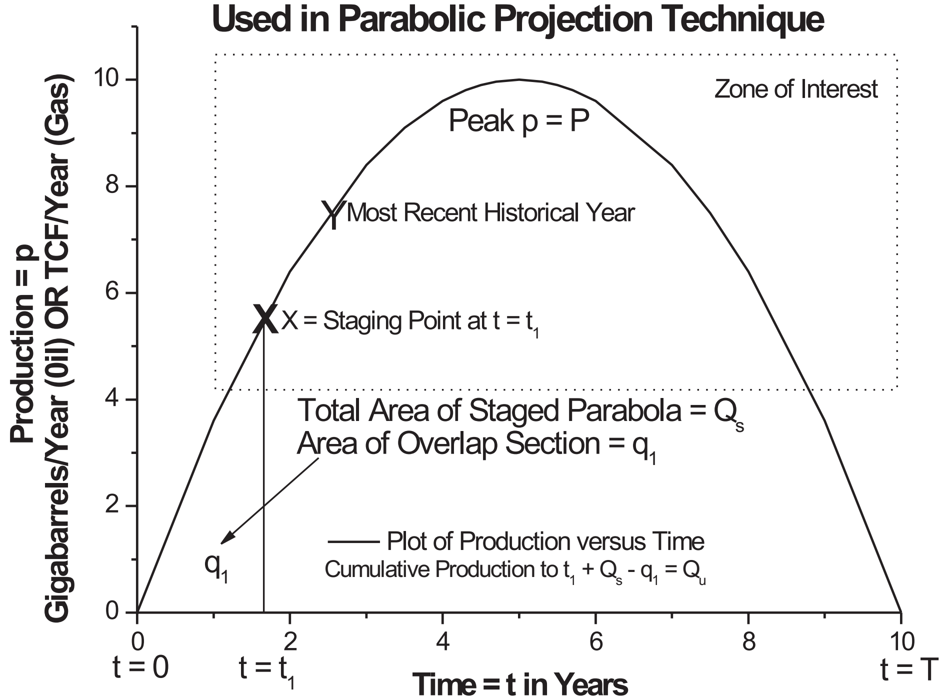
Introduction

The parabolic technique for the projection of conventional oil and gas resources was devised to interpret the results of geological assessments in terms of future production over time. Though a parabolic function is employed, the resulting plots are not Hubbard Curves in the sense that their shape does not depend upon the history of the production of oil or gas except for the most recent decade. The projection is instead an alternative to the usual straight-line interpretation in terms of reserve or resource to production ratios in that it gives rise to a peak after which production declines. The parabolic technique is thus an aid to the interpretation of assessments of conventional oil and gas resources, not an alternative system for determining the size of the resource. It is therefore not a `neohubbertarian' appoach in the sense used in a recent concise but useful summary of developments in the field by Hall *et al*.[1]

The parabola was chosen for the modelling equation because it is the simplest mathematical function displaying a peak that is not constrained by inflection points on each side of the maximum as, for example, are logistical or normal curves. The absence from such inflection points allows the zone around the peak to change more smoothly and more slowly than other mathematical functions. The parabolic function is also more versatile in that it permits easy-to-handle variations to deal with such complications as the reserves addition, the existence of idle capacity, and plateau peaks, all of which are more difficult to model in other approaches. The fact that a parabola crosses the time (or X) axis at two definite points is helpful in conducting the mathematical manipulations but this curve will not model the extremes in time when production begins slowly on one side and tapers off gradually on the other as will logistical or normal curves. Because only the most recent past decade or so is relevant to the computation, this failure to develop early approaching 'tails' is of no significance. Only the 'tails' of the far future on the other side of the peak are not well handled by this method but this period is rarely of great interest. The main focus here is on modelling the region around the peak as shown in the zone of interest delineated in Figure 1.

The purpose of this paper is to illustrate the method in a clear way to allow others to use it without difficulty. A hand calculator is all that is required although the use of a standard mathematical computer program will ease the procedure at one stage. After setting out the details of the method, examples of its application are taken from recent papers and notes to illustrate the solutions developed to handle the special problems encountered.

Figure 1: Illustration of Terms Used in Parabolic Projection Technique



Methodology of the Parabolic Projection Technique

The Basic Parabolic Relationships

For a parabola of the type illustrated in Figure 1, the underlying equation may be written in the following way because the usual quadratic 'c' term is zero in this case:

$$p = at^2 + bt$$

When p = 0, t = T and so a = - b/T: b may then be expressed in terms of Q, the total area of the parabola, by integration from 0 to T:

$$Q = p\int dt = -b/3\ T^2 + b/2\ T^2;\ \text{thus}\ b = 6\ Q/T^2$$

The production equation then becomes by substitution:

$$p = 6Q/T^2\ t\ (1 - t/T) \qquad \textit{... Equation 1}$$

If r is defined as t/T, the equation becomes

$$p = 6Q/T\ r\ (1 - r) \qquad \textit{... Equation 2}$$

Equation 1 may then be integrated to obtain the partial area of the parabola, q, up to the time, t. After re-arranging, the following result is obtained:

$$q/2Q = 1.5\ r^2 - r^3 \qquad \textit{... Equation 3}$$

The special case of the peak P at time T/2 inserted in Equation 1 results in the classic formula for the area of a parabola:

$$Q = 2/3\ PT \qquad \textit{... Equation 4}$$

Assessing the Value of Q_u

The term Q_u is taken as the *ultimate recovery of the resource* which is frequently designated as *URR* in other papers in the field. It represents the total area under a production-time curve from the start to the finish of the exploitation of a resource. A geological assessment of undiscovered resources will apply to a specified year. Typically, a mean value is given along with 90% and 10% probability limits. A Reserves Addition may be specified or not.

For the year of the assessment, Q_u is calculated by adding the cumulative production to date to the already established reserves to which sum is added the assessment of undiscovered resources. The Reserves Addition, if available, is not included in Q_u at least initially. The value for the established reserves is published each year in reputable publications but is generally a subject of considerable controversy. The cumulative production is less frequently published and its value must often be adjusted to the year of the assessment by adding or subtracting published yearly production figures to arrive at the desired time. Separate cases are calculated for the mean value and the two extremes of the assessment.

In a perfect world with the resource well understood, this total would be independent of time. In practice, the value of Q_u, both for the world in total and for most of its producing provinces, has been increasing with time as improved exploration techniques spread around the world. (There have been cases where the value has declined as for the eastern offshore developments of Canada). For this reason, it is desirable to use cumulative and reserve data that is as consistent as possible in time with the date of the geological assessment.

Selection of the Staging Point

The parabola to be calculated is termed a Staged Parabola here because it starts at a Staging Point usually chosen to be ten years earlier than the most recently available production data. At the Staging Point, both the production in that year and the cumulative production to that time are known. It follows that the ultimate total production (Q_u) – the total production of oil from the historical start to finish - equals the sum of the cumulative production to the staging year (t_1) plus the area of the whole of the Staged Parabola (Q_s) less the area (q_1) of that portion of the parabola that overlaps the cumulative production up to time = t_1 as illustrated in Figure 1. In this equation, two quantities are not known: the area of the Staged Parabola (Q_s) and the area of the overlap section (q_1).

Method of Iterative Solution

As an analytical solution to these equations could not be found, an iterative method had to be devised. To start, a likely value for the ratio of q_1/Q_s is assumed. The normal practice for iterative calculations is followed in that the particular solution or a given trial lies between 0 and 1 and is usually in the range 0.15 to 0.2. With the assumption of the value of this ratio, given that the value for Q_u has been calculated previously as noted above and that the cumulative production is known to the Staging Date, numerical values may be obtained for q_1 and Q_s.

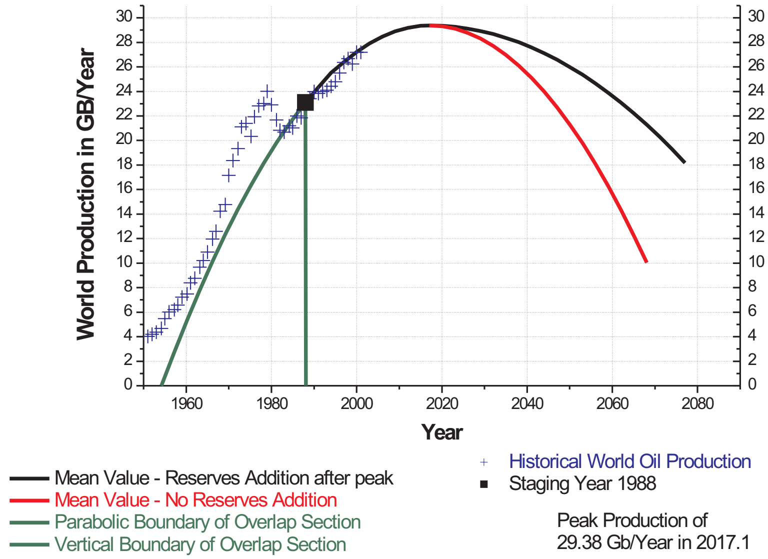
The value of $q_1/2Q$ is entered into Equation 3 which is then solved for r_1. This solution may be found by hand iteratively although it is desirable to have five significant figures past the decimal point. Given a library of past solutions to the function ($1.5 r^2 – r^3$) such accuracy may usually be obtained in eight to twelve trials. A mathematics program is useful in avoiding this time-consuming step. For low-end programs, it is helpful to introduce $R = 100r$ before solving for R and then dividing by one hundred to achieve the necessary accuracy.

With knowledge of r_1, it is possible to determine a value for T using Equation 2 because Q_s and p are known for each particular iteration. The value for t_1 can then be determined with t_2 normally ten years later. The value of r_2 is obtained from t_2/T.

Testing the Iteration

For the case of oil, world output normally increases in a consistent and stable manner. Only infrequently does production fall below that of the previous year as it did in 1992 according to some statistical sources. Given this regularity, the simplest test of the iteration is whether the value of r_2 as derived above in each trial predicts the correct production in the most recent statistical year when introduced into Equation 2. Successive iterations with different assumed values for q_1/Q_s should be continued until the predicted value equals the actual production ideally to two decimal places. It is important that the iterations approach the solution from both sides because the calculation is essentially nonlinear and may proceed only very slowly to an acceptable solution. As is the usual practice in iterative trials, after the final result has been straddled, it

Figure 2: U.S. Geological Survey World Mean Case Reserves Addition after Peak Production



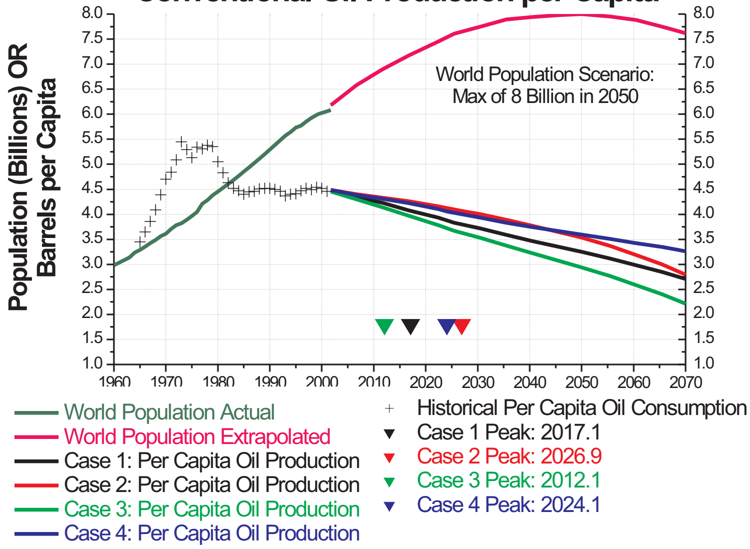
is helpful to plot each production prediction as it is obtained versus its variation from the actual production value to determine the value for q_1/Q_s from the zero crossover point for use in the next trial. With experience, a solution of the desired accuracy can usually be obtained within twelve attempts.

The final solutions for Q_s and T are then introduced into Equation 2 to draw the parabola with t the variable over the range desired. The starting point may be located on the graph of production (Y axis) versus years (X axis) by deducting t_1 from the Staging Year. This method of testing the iteration has the advantage that the parabolic curve passes through the point for the most recent historical production date. A quick estimate of peak production is possible by introducing Q_s and T into Equation 4. The year of the peak is determined by adding T/2 to the starting date.

This approach is less satisfactory in the case of natural gas because this fuel is generally earlier in its life cycle than oil. As a result, at least up to now, the fluctuations in annual production tend to be greater than for oil. The production of natural gas in the most recent year may be less than in the immediately previous year. A simple test is possible to decide whether the more complicated and time-consuming method described below is necessary. The actual production is plotted by year over the ten-year period between the selected Staging Point and the most recent statistical year. The area under this curve is then estimated as the sum of a rectangle and a right-angled triangle drawn between these two extreme years. This value is then compared with the actual cumulative production between these two dates. The actual cumulative value should be slightly higher than the sum of the two geometrical figures. If the divergence is more than one per cent, it is better to use the following alternative method for testing the iterations.

The procedure is the same as the oil case above to determine r_1 and r_2 for each iteration. This data is then used instead to calculate the partial area of the parabola extending from the start at $t = 0$ to t_1 and then t_2 designated as q_1 and q_2 respectively. The difference between $q_2 - q_1$ is then compared with the actual cumulative production between t_2 and t_1 for each iteration. The desired solution results when this difference is reduced to zero to two decimal places. This approach is more accurate that the previous technique described in the first procedure that depends upon matching the predicted production in the most recent year to its actual value. The disadvantage is that the plotted parabolic curve does not necessarily have to pass through the most recent historical production point. There is no reason this second procedure could not be used for the oil cases as well if desired.

Figure 3: World Population and Conventional Oil Production per Capita



Special Cases

The Reserves Addition

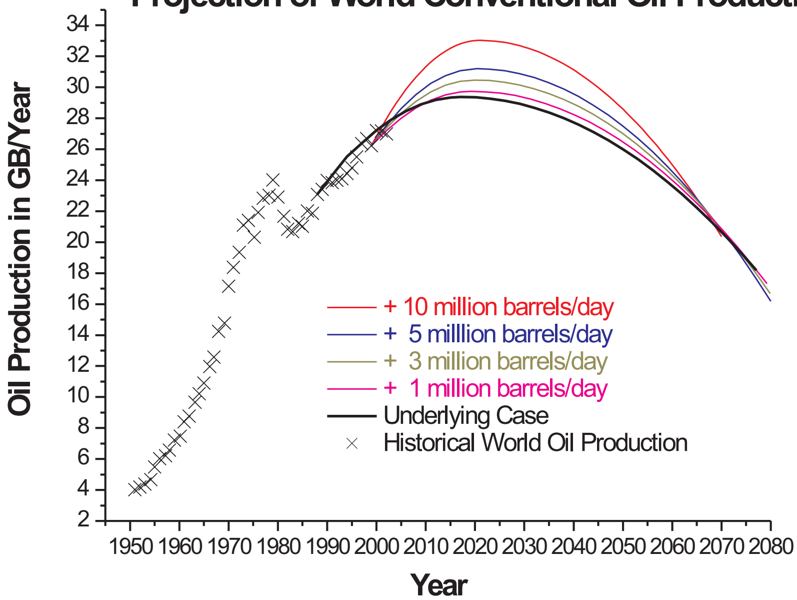
Geological resource assessments often specify a Reserves Addition to reflect the growth in reserves frequently experienced at a discovered reservoir over time as distinct from estimates of greater undiscovered resources. This subject is controversial but this paper is concerned only with the manner of representing this reported growth in terms of potential production in the parabolic plots.

At one extreme, the reserves addition could simply be added to the estimate of undiscovered resources and thus ultimately to Q_u to serve as the basis of the parabolic projection. This procedure implies that the reserves addition contributes uniformly to the production over the life of the reservoir. This is unlikely because this value is probably quite price sensitive. The opposite extreme is to assume that the reserves addition only becomes effective after the peak is passed. The rationale in this case is that the higher prices to be expected after the peak provide an added incentive to apply the more costly and complex measures needed to increase the recovery of oil from the reservoir. The truth may well lie between these two extremes. (The problem is generally not encountered in the natural gas field because recoveries in normal practice are higher than in the case of oil.)

When the reserves addition is assumed to become effective only after the peak has passed, a new extended parabola may be drawn which shares its value and timing of the peak already calculated for the underlying case. Twice the reserves addition is added to Q_s to determine the new parabola whose area is designated Q_e. This is because all the reserves addition is required to appear on only one side of the extended parabola to determine the new value of the production after the peak. In this case, Equation 4 is used to determine T_e from Q_e given the value of the shared peak, P, taken from the underlying parabola. An extended parabola may then be drawn from the peak onwards in time by using Equation 1 employing the values calculated for Q_e, and T_e as far into the future as desired.

The reserves addition may also be split between the underlying and extended parabolas. It may be decided as a matter of expert opinion that the reserves addition should be divided as one-quarter occurring before the peak and the remainder thereafter. The one-quarter fraction would be added to Q_s and the iterative procedure followed to a solution for the underlying case. The extended parabola would then be determined by adding twice the remaining three-quarters fraction of the reserves addition to Q_s to determine Q_e and thus T_e via Equation 4. Any division of the reserves addition may be accommodated in this manner in the light of experience. This is one of

Figure 4: Effect of Idle Capacity on Parabolic Projection of World Conventional Oil Production



the strengths of this methodology but, as a practical matter, calculation of the two extreme cases is usually sufficient.

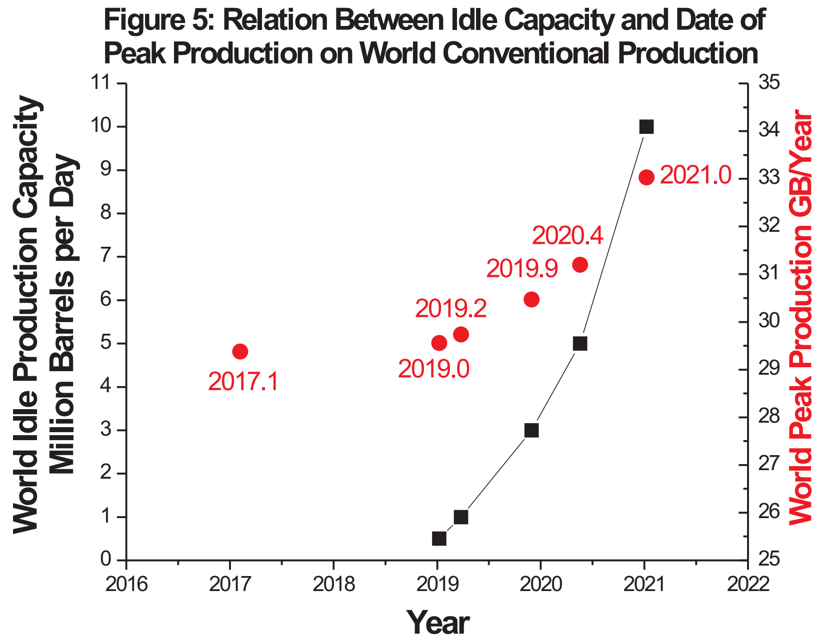
An example of this procedure appears in Figure 2 concerning the parabolic projection of world oil resources as determined for the world assessment of the U.S. Geological Survey published in 1990.[2] In Figure 2, the reserves addition is treated as only effective after the peak is past. In this technique, there is no reason for the historical production to lie on the parabolic curve in the distant past. It is only the period near the peak and shortly thereafter that is modelled.

The results of the application of this procedure to all four cases for the U.S. Geological Survey assessment as calculated in Reference 2 are plotted in Figure 3 in terms of per capita production. In Case 1, with the reserves addition becoming effective only after the peak has passed, world conventional oil production was predicted to peak at 29.38 gigabarrels (GB) per year in 2017.1; in Case 2, with the reserves addition effective all throughout the production period and so simply added to Qu, the peak was 31.1 GB/year in 2026.9. In the two sensitivity Cases 3 and 4, the reserves addition was assumed effective only after the peak as in Case 1. In Case 3 at 95% probability, the peak was 28.53 GB/year in 2012.1; and in Case 4 at 5% probability, the peak was 30.60 GB/ year in 2024.1. World per capita production was calculated for these four cases employing a scenario characterized by a conservative peak in world population of eight billion in 2050. It will be seen that in all four cases, the world per capita production falls from the present on. This decrease is true over the wide range of resource assumptions specified for the four cases.

It will also be noted that the world has already passed its peak in world per capita consumption (over a full year this value is essentially equal to production) and that this value has remained remarkably constant for the last two decades.[3] The plots in Figure 3 indicate this empirical relationship will end in the near future.

The Problem of Idle Capacity

All methods of projecting resource assessments are faced with a major difficulty when there is idle capacity in the world conventional oil production system as there is at present. For most commodities, idle capacity occurs during periods of economic recession but in the case of oil, potential production may be held off the market to support prices or to meet the political objectives of the OPEC group of nations. Given a normally functioning and reasonably com-



petitive marketplace, the present established reserves are sufficient to support more output than has actually been the case. With lower prices, production would have been higher at both the time of the Staging Point and the most recent statistical year and thus so would have been the predicted peak production.

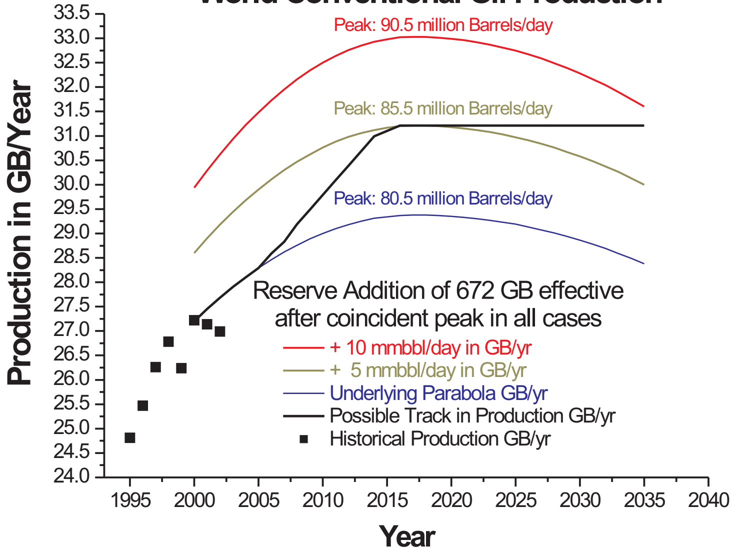
There are two related problems is dealing with this situation. First, there is the uncertainty as to how much actual idle capacity there is throughout in the world. In this paper, it is assumed that idle capacity refers to non-operational existing oil production facilities that could be restored to service within a year or so. The larger question of the extent to which installed facilities could be expanded to exploit the already discovered reserve base is beyond the range of this paper. With this more limited definition, the idle capacity is probably between three and ten million barrels per day with most of it located in the Middle East.

The second uncertainty is the rate at which this idle capacity is likely to be brought back into production. Here it is assumed that except for Saudi Arabia and perhaps one or two of its close allies and neighbours, the idle capacity will be restored to service by the time the peak in the world production of conventional oil is reached.

Both of the solutions offered here to deal with this problem involve adding the subjective estimates of idle capacity to the peak calculated for the underlying parabola that appears in Figure 2. Implicit in this procedure is the assumption that the simple addition does not itself affect the timing of the peak sufficiently to make the additive procedure invalid. The more rigorous of the two approaches involves repeating the iterative procedure used to determine the underlying parabola but applying it separately to each of the plus one, plus three, plus five, and plus ten million barrels per day estimates of the idle capacity added to the peak previously determined for the underlying case. The procedure followed is the same as before except that the test for the desired iterative solution is the value of the new peak that incorporates the level of idle capacity assumed for each case. A new parabola is then determined which passes through this peak. When this procedure is followed, it is no longer necessary to use the same Staging Point for the new idle capacity parabolas as was used to compute the underlying parabola. It is more accurate to shift the Staging Year for these additional parabolas to the most recent historical data year available though nothing prevents the use of the original Staging Point.

In practice, a particular level of idle capacity is assumed. This is added to the peak already determined for the underlying parabola. The Staging Point is

Figure 6: Alternative Treatment of Idle Capacity in World Conventional Oil Production



normally chosen at the most recent historical production year available which is usually ten years more recent than the original Staging Point employed in the calculation of the underlying parabola. Values for Qs and T are obtained for each iteration of the ratio q/Qs as before. Qs and T may then be inserted into Equation 4 to determine the corresponding value of the new peak, P. Iterations are continued until the predicted value for the peak equals the desired P which is the sum of the original peak on the underlying curve plus the particular assumed level of idle capacity. The right hand side parabola is of the extended type to account for the reserves addition which is assumed in this case to be only effective after the peak using the technique described in the previous section. The results of four parabolas for world idle capacities ranging from plus one to plus ten million barrels per day appear in Figure 4.

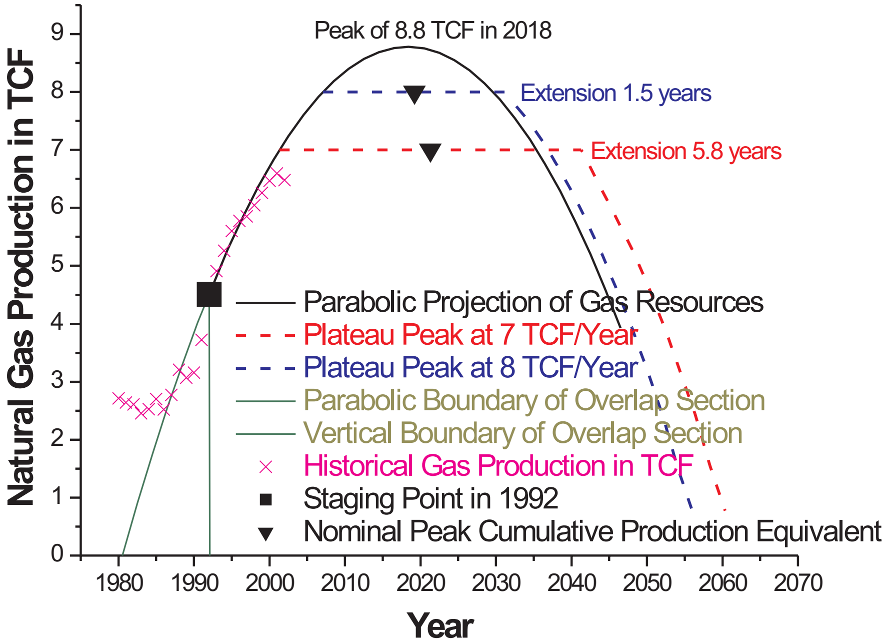
The timing of the peaks calculated in Figure 4 are plotted in Figure 5 plus one extra case at plus 1.5 million barrels per day. (This latter case is not plotted in Figure 4 as it is too close to the underlying case to distinguish on the graph.) The peak shifts almost exactly four years later from the underlying case to the maximum case of plus ten million barrels per day idle capacity. This calculation is elaborated in more detail in Reference 4.

Given the relatively small shift in the timing of the peak as the idle capacity is increased and considering the other inherent uncertainties, a simpler but less rigorous procedure may be all that is justified. In Figure 6, each of the parabolas are drawn by employing Equation 4 to arrive at new values for T at the same Qs for the different values of P calculated by adding the various assumed levels of idle capacity to the peak obtained in the underlying case. The peaks are all coincident in time in this simpler procedure. Care is taken to prepare matching extended parabolas on the right hand side for this case when the reserves addition is only assumed effective after the peak as before.

An application of this technique is also illustrated in Figure 6. If it is assumed the total idle capacity is ten million barrels per day but that only five million barrels per day will be restored to service by the time of the peak, it is possible to draw a possible track of oil production through this period. The actual production is shown as gradually rising from the underlying parabola to account for the five million barrels per day to be restored to service by that time. The production then stays constant at this peak level until the line intersects the outer parabola at plus ten million barrels per day during the period when the remaining five million barrels per day of idle capacity are absorbed before the final decline begins.

The advantage of this latter method is its simplicity in providing insights into what is very likely to occur in practice. The technique implies there will be a period of some years of 'plateau production' in world

Figure 7: Parabolic Projection of National Energy Board `Supply Push' Case Assessment of Natural Gas Resources



oil production before final decline begins given these assumptions. The effect of changes required in the light of experience may be calculated easily.

Plateau Peak Cases

In the case of natural gas, especially in North America, a flat extended peak is expected which is termed here a 'plateau peak'. There are two reasons this type of production profile is likely to occur. The first arises from the nature of most gas reservoirs. Because the recoveries from natural gas reservoirs are generally greater than from corresponding oil cases, there is an incentive to keep production constant as long as possible from a given source rather than contend with rapidly increasing and then quickly decreasing output. The second reason concerns the location of the remaining conventional resources of natural gas in North America. These will tend to be found in northern or other inhospitable offshore regions requiring expensive production facilities as time goes by. Delays are to be expected in bringing this gas to market so production may depend more on the deployment of facilities than the discovery of resources.

The data taken from the Appendix of the *Supply Push Case* formulated in the 2003 National Energy Board report on supply and demand was used to draw the parabola in Figure 7[5] using the iterative procedure with the Staging Point in 1992.[6] There is no extended parabola in this case as is typical for natural gas cases. Though the peak is predicted to be 8.8 TCF in 2018, the Board's analysis of the individual components of the supply leads them to expect production to level off between seven and eight trillion cubic feet per year. The plot of historical production also suggests the formation of a plateau peak. To accommodate this view, horizontal lines are drawn at these two levels on the parabola. These are best located accurately by solving the quadratic equation of the parabola for the two opposite values for t on each of its sides for the 7 and 8 TCF/Year cases in turn. The area of the unproduced top section of the parabola may then be calculated from Equation 4 for each case. The length of time for each horizontal extension may then be determined by dividing this area by the value of its relevant production plateau. The area of the rectangle bounded at the top by this extension is thus set at the area of the cut off peak for each case. When this quantity of gas is exhausted, the decline sets in according to the parabola displaced out in time by the extension period. It may be shown that the longer the extension period of the plateau peak, the faster will be the decline once it begins.[7]

The nominal time to reach peak production may be defined as the year when the cumulative quantity of gas produced during the plateau period equals that which would have been produced up to the time of the peak of the unconstrained parabola. This may simply be calculated as the year of the peak of the parabola to which is added one half of the time of the respective extensions. In Figure 7, for a plateau of 7 TCF/Year, the nominal year of peaking is plotted at 2018.4 + 5.8/2 = 2021.3 and for 8 TCF/Year, at 2018.4 + 1.5/2 = 2019.2.

Conclusion

The continuing controversy and concern related to the extent of the world's undiscovered conventional oil resources and the North American natural gas situation has led to the development of the parabolic projection technique. At the heart of this approach is the expectation that there will be frequent assessments made by geological experts in this field over the next decades. The parabolic projection technique was devised to permit production versus time curves to be drawn to make better use of these assessments in a coherent way to permit more meaningful interpretations. The methodology is reported in sufficient detail in this paper to allow others to employ and assess calculations of this type.

From the examples chosen to illustrate this technique, it is difficult to escape the conclusion that the world production of conventional oil will peak about 2020 and that North America is faced with a 'plateau peak' in conventional natural gas production in the near future, if this latter situation has not arisen already.

The technique is also useful in the field of climate change. Due to the low technical cost of production of most conventional oil and gas resources, the tendency to produce them will be high. The resulting carbon dioxide emissions over time may be calculated readily from the parabolic projections.

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