Comparison of the Deffeyes Plot with the Staged Parabolic Technique for the Prediction of the Peak in World Production of Conventional Oil

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Abstract

The plotting technique used by K.S. Deffeyes in a recent book was applied to a current assessment of future world production extending through to 2050 prepared by the *Association for the Study of Peak Oil* (ASPO). The ultimate cumulative production of conventional oil (broadly defined) is predicted to be 2300 gigabarrels (GB). This technique was then modified to predict the magnitude and timing of the peak in the world production of conventional oil by incorporating resource assessments published by the U.S. Geological Survey in 2000. The results were compared with the previous predictions based upon the Staged Parabolic Technique reported in an earlier paper. The values calculated for the magnitude of the peak from the Deffeyes Plot were found to be in reasonable agreement but the timing was found to occur about three years earlier when the parabolic method was employed.

In his recent book, K.S. Deffeyes estimated the total ultimate world production of conventional oil using a derivative form of the logistic or 'S' curve.[1] The derivative traces a parabola and when the production approximates this function with respect to the cumulative output, the equation may be written in the following form:

$$p = aq^2 + bq = aq^2 - aQq$$
since $q = Q$ when $p = 0$

This equation may be simplified to a straight line when p/q is plotted against q as follows:

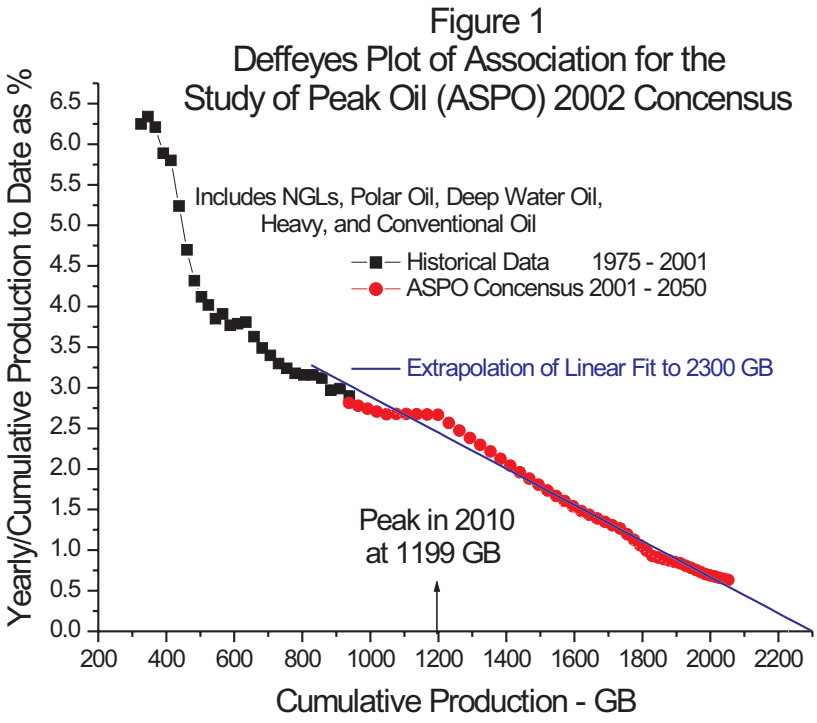
$$p/q = aq - aQ \quad \text{or} \quad p/q = -a\,(Q-q)$$

where p and q = annual and cumulative production at a given point, and Q = ultimate cumulative production.

The main advantage of such a plot is that a downward-sloping straight line results. This line may be easily extrapolated to the q axis at which intercept it will give the value of Q directly. Other modelling equations that might be used in place of the parabola, such as the normal curve, will approach this straight -line behaviour on this plot as maturity is reached. In practice, it has become the custom to express the ratio of the annual production to the cumulative production for each year in terms of a percentage value. Laherrère has commented on this derivation and more generally on the limitations on the use of logistic curves for this purpose.[2]

The Association for the Study of Peak Oil (ASPO), at its May 2002 Meeting at Uppsala University, Sweden, reached a consensus among its participants as to the outlook for the production of a widely defined range of oil types extending to 2050.[3] The peak is expected as soon as about 2010 as may been seen in the production-time graph appearing in the Appendix. This plot was converted to a Deffeyes-type plot in Figure 1. Production statistics for the twenty-seven year historical period from 1975 to 2001 were taken from the *BP Statistical Review of World Energy.*[4] The base point for this calculation was a carefully-estimated value for cumulative oil production of 610.1 gigabarrels (GB) for the world up to the end of 1988 published by Masters et al.[5] The cumulative production was computed by adding or subtracting yearly production in sequence in both directions from the 1988 reference point.

The yearly data taken from the published ASPO production-time plot from 2001 to 2050 was converted year-by-year into the p/q versus q form required for the Deffeyes Plot. A computer-generated best fit line was then extrapolated to the q axis to determine a value of the ultimate total production Q of 2300 GB. This value is almost exactly the value of the 95% Probability Case of the U.S. Geological

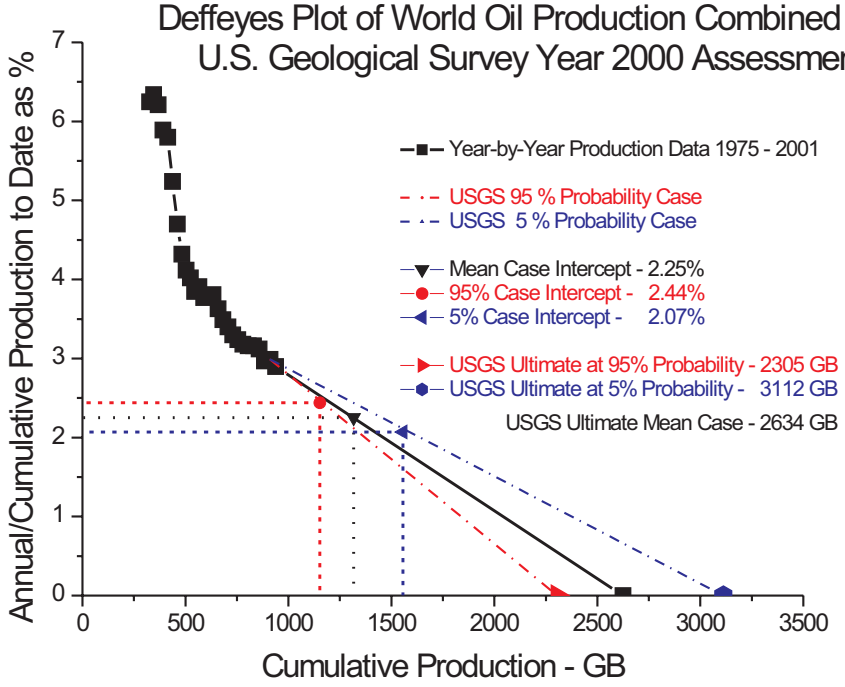


Survey assessment used later in this note.[6] Nevertheless, a careful examination of the plot at the extreme right of the ASPO range (the data most far out in time) suggests this value would have been even higher if the extrapolation had been based only upon the last sequence of points. Peak production would be expected at one-half the predicted total endowment of 2300 GB at 1150 GB rather than the 1199 GB actually derived from the ASPO plot suggesting the latter is not completely symmetrical.

The historical section of the plot in Figure 1 (repeated in Figure 2) agreed with that published by Deffeyes in his book[1] though it was not clear what statistical source that author had used. His extrapolation indicated the world endowment of conventional oil was about 1800 GB - much less than the 2300 GB predicted by the extrapolation of the ASPO data. It may be that Deffeyes was using a narrower definition of what constitutes conventional oil. However, it may be seen by inspection of the graph in Figure 2 that though the straight-line section could be extrapolated to 1800 GB on the horizontal *q* axis, higher values are also possible. Even if a best-fit line were to be computed electronically from the historical data, there would still be the question as to which points should be selected as components of the straight-line segment. In this author's view, there is not sufficient mature data available at present to extrapolate this line from the historical record with any great confidence in its accuracy.

Given this uncertainty, the Deffeyes Plot was used in a quite different way in this note. Rather than making a doubtful extrapolation to estimate the ultimate cumulative production of conventional oil, this technique was used instead to estimate the peak production by introducing the world resource assessment of conventional oil published by the U.S. Geological Survey in 2000.[6] This estimate of the quantity of conventional oil yet to be discovered is provided by the U.S.G.S. in the form of a Mean Case at average expectation bounded by a low value at 95% probability and a high value at 5% probability. In the view of Laherrère, however, these values are 'implausibly' high.[2] The world's ultimate endowment of conventional oil was then estimated by adding the cumulative production and the published reserves at the time of the assessment as was done in the previous paper on the Staged Parabolic projection technique.[7] Many experts believe the published value for world reserves is also overstated for a variety of economic and political reasons. The value for the Mean (2634 GB) obtained in this way is mark-

Figure 2
Deffeyes Plot of World Oil Production Combined with
U.S. Geological Survey Year 2000 Assessment



edly higher than the 1800 GB predicted by Deffeyes and higher than the 2300 GB predicted from the ASPO data.

Whatever the true value of Q, the same geological assessment was used in the previous paper on the staged parabolic method and thus the results determined from the two methods may be compared. The importance of knowing the magnitude and timing of the peak in world conventional oil production was explored in detail in Reference 8.

The three values for the total ultimate cumulative oil production based upon the U.S. Geological Survey data were plotted on the horizontal q axis in Figure 2. Lines were then drawn from the last point on the world historical data series to these ultimate points on the q axis as shown on the graph. The outer lines represent the 95% (low) and 5 % (high) probability boundaries to the Mean Case. As the peak production was assumed to occur at the mid-point of the total oil endowment for each of the three cases, lines were drawn vertically from the half-way points on the q axis so as to intercept the plot of the percentage of cumulative production. The corresponding values on the p/q axis opposite each intercept were then multiplied by the respective cumulative values at the half-way point to estimate the peak annual production.

	U.S.G.S. Mean Case		**U.S.G.S. 95% Prob.**		**U.S.G.S. 5% Prob.**	
	Parabola	**Deffeyes**	**Parabola**	**Deffeyes**	**Parabola**	**Deffeyes**
Peak Product. GB/yr	29.38	29.64	28.53	28.16	30.60	32.28
Peak Year - Ref. 6	2017		2012		2024	
Peak Year - Rigorous		2014		2009		2021
Peak Year - Approx.		2014		2009		2022

The values of the peak production may also be obtained by manipulating the properties of the lines drawn in the figure using the boundary conditions at each end of the straight-line sections. Since the peak production, *P,* is assumed to occur at the half-way point of the ultimate cumulative production, this value may be calculated from the following equation:

$$P = pQ^2/4q(Q - q)$$

where P = peak production, p and q = annual and cumulative production at last historical point and Q = ultimate cumulative production, as before, but for each of the three cases.

The peak production computed in this way is listed in the table and compared with the peak values taken from the earlier paper based upon the Staged Parabolic Technique.

The estimation of the timing of the peak poses a problem using the Deffeyes plotting technique. The approximate method is to average the production in the most recent historical year (here 2001) with that estimated for the peak. Since the difference in the cumulative production between these two times is known from the plot, the year of the peak may be estimated by dividing this difference by the average production to determine the time from 2001 to the half-way point.

A more rigorous calculation was also made which depends on the parabola which underlies the logistic equation. This parabola is the trace of the derivative of the logistic equation with respect to time. It is possible to apply the equations used in Reference 7 to this parabola to estimate the timing of the peak. The details of this calculation may be obtained from the author. It will be seen from the agreement between the two approaches in the table that there is little reason to use the more complicated rigorous technique.

The results calculated here from the Deffeyes Plot are in good agreement with respect to the output of oil expected in the year of the peak for two of the cases with those found in the earlier paper based upon the Staged Parabolic Technique. There was fair agreement for the third case at 5% probability. The results with respect to the timing of the peak are less satisfactory with the peak in the Deffeyes approach predicted to occur some three years earlier than in the parabolic technique. The reason for this difference is not clear but may be because time is not an explicit variable in the case of the Deffeyes Plot. The time estimate may be more uncertain on this account.

The main advantage of the Deffeyes Plot is that once prepared, it may be updated in a matter of minutes as new production data becomes available each year. If and when updated geological assessments are published, the new data points may be plotted easily on the *q* axis. Visual comparison to judge the significance of conflicting geological assessments may also be made quickly by plotting on the *q* axis. The Deffeyes Plot is thus an excellent way of tracking progress as time passes.

The main disadvantage of the Deffeyes Plot is its limited flexibility. Unlike the Staged Parabolic Technique, adjustments for changes to reserves (as opposed to resources) are difficult if not impossible to make over specified time periods. The same is true of the main difficulty afflicting all methods of projection based upon historical production data —there is no easy way of dealing effectively with the excess idle production capacity that exists in the world at present. Oil production is less than it should be for both economic and political reasons. If in operation, this extra production would have the effect of increasing the value of the peak production predicted. In contrast, the Staged Parabolic Technique offers a way of compensating for this effect.[9] The peak predicted by the Deffeyes Plot will thus tend to be a little low as long as the present idle capacity persists.

The ease with which a Deffeyes Plot may be updated makes it an attractive first step in following the evolution in the prediction of the magnitude and timing of the peak in the world's production of conventional oil.

References

1. K.S. Deffeyes, *Hubbert's Peak: The Impending World Oil Shortage*, Princeton University Press, 2001. ISBN 0-691-09086-6.

2. Jean Laherrère, *Comments on the Book 'Hubbert's Peak: The Impending World Oil Shortage*, (Web: www.hubbertpeak.com/laherrere/Deffeyes_comments.pdf.)

3. Meeting of the *Association for the Study of Peak Oil* (ASPO), Uppsala University, Sweden, May 23-24, 2002. Abstracts of Presentations and the Press Release which includes the consensus graph on future oil supply may be found on the Web at *www.isv.uu.se/iwood2002*. An archive of newsletters published by the group may be found at *www.energiekrise.de* under the ASPO tag.

4. *B.P. Statistical Review of World Energy*, published yearly in late June by BP p.l.c., London, England. (Web: www.bp.com/centres/energy2002)

5. C.D. Masters, D.H. Root, and E.D. Attanasi, *Resource Constraints in Petroleum Production Potential*, Science, Vol. **253**, 12 July 1991, 146-152.

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7. J.H. Walsh, *Parabolic Projection of World Conventional Oil Production Based on the Year 2000 Resource Assessment of the U.S. Geological Survey*, Proceedings of the Canadian Association for the Club of Rome, Series **2**, No.3 Spring/Summer 2001.
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8. J.H. Walsh, *The World Energy Situation after the Peak in Conventional Oil Production has Passed*, Proceedings of the Canadian Association for the Club of Rome, Series **3**, No. 3, March 2002.
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9. J.H. Walsh, *Parabolic Adjustment for Idle Capacity in the World Conventional Oil Production System*, September 2000.
(Web: pages.ca.inter.net/~jhwalsh/wusgsi.html)

Revised and updated with 2001 production statistics in July 2002.

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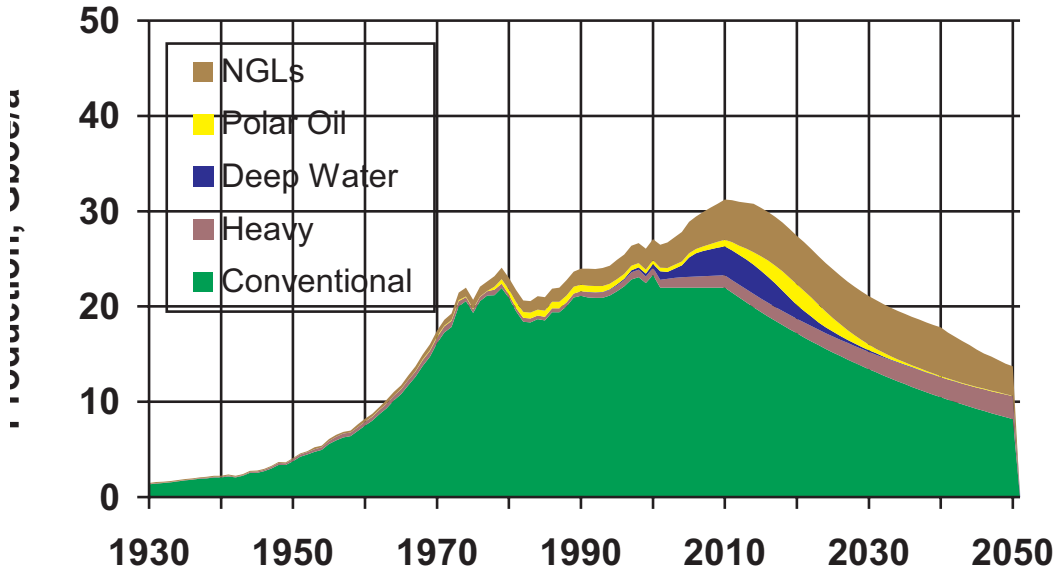
APPENDIX

Association for the Study of Peak Oil (ASPO) Press Release May 2002 Web: www.isv.uu.se/iwood2002

The Impending Decline of Oil Supply

A meeting of ASPO was held in Uppsala University on May 23rd and 24th under the chairmanship of Professor Kjell Aleklett



to address the subject of oil supply.
Speakers from Sweden, Norway, Denmark, United Kingdom, Ireland, Germany, France, Portugal, the United States, Russia and Australia discussed this important topic.

The seminar was the foundation meeting of ASPO, a group of European universities and government institutions.

The world oil depletion curve, above, is based on all available information on oil reserves and estimates of the amounts yet-to-find, and indicates that world oil production will reach a peak around 2010 and decline thereafter. The seminar evaluated the evidence for this forecast, and addressed the important political and environmental consequences.

ASPO plans to update the evaluation every year as new information and insights come in, with the intention of providing governments with a reliable basis for planning their responses to this critical issue.
(Production expressed in GB oil equivalent on y-axis.)