Note on Compound Decline Curves for World Conventional Oil Production

Web: pages.ca.inter.net/~jhwalsh/wcompounddecline.html

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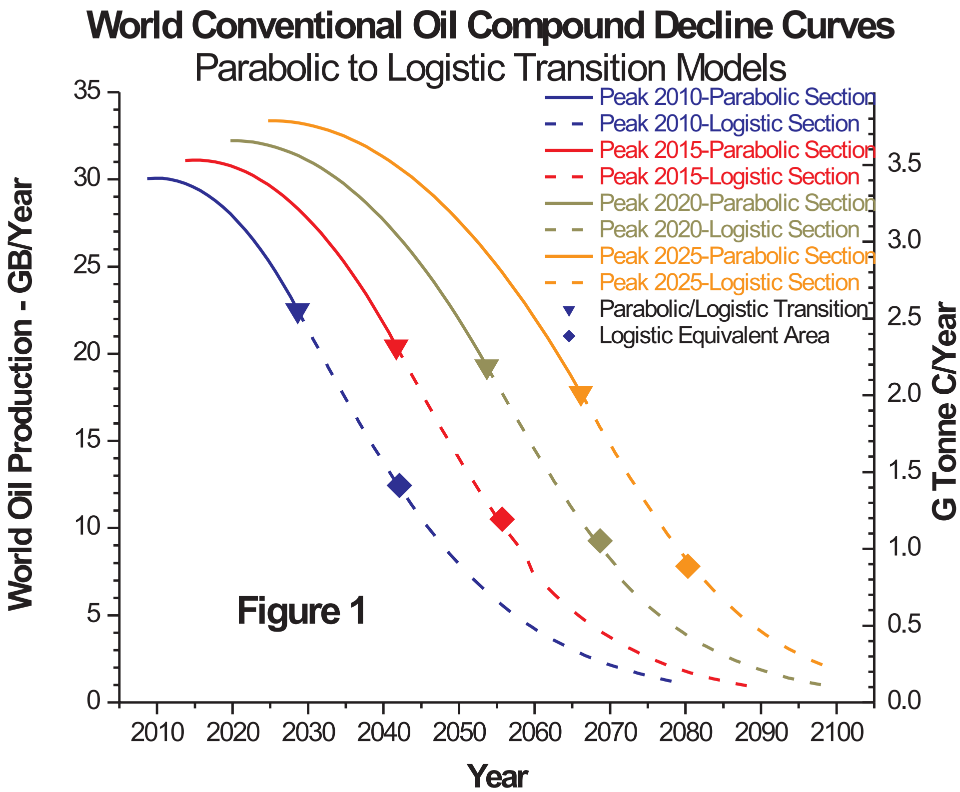
The parabolic technique for projecting oil supply was devised mainly to model conditions near the peak.[1] Though other curves may be applied for this purpose, the parabola has the advantage that there are no points of inflection on each side of its maximum to constrain the form of the peak over the range of main interest. It is thus possible to obtain a smoothly changing line that in extreme cases might even model a situation approximating a plateau instead of a peak. This feature is important when the world as a whole is to be modelled, as opposed to an individual basin or country, because the sum of all producing basins, each at different stages of maturity, is likely best modelled by a relatively flat, smooth curve. In contrast, for individual cases, especially those where the bulk of the production is from off-shore platforms such as in Norway or the U.K., the production curve tends to fall from a sharper peak due to the higher rates of depletion generally experienced. The disadvantage of the parabolic curve arises from its fixed interception of the abscissa so that the decline period will be modelled increasingly poorly as the distance past the peak increases. Although the quantity of oil under the curve becomes smaller and less important in an energy sense with time after the peak, the decline period is of growing interest in an environmental sense when future carbon dioxide emissions must be estimated.

The evidence to date suggests that the period well past the peak is best modelled by some form of logistic curve.[2] To accommodate the need to model this later period in the decline, a method for converting parabolic curves smoothly to logistic form is presented in this note. This technique was applied to a series of four peaks in world conventional oil production assumed to occur at five-year intervals over the range of interest from 2010 to 2025. The slope of the parabolic curve was determined at the transition between the parabolic and logistic sections of the curve with the logistic section starting out at this slope and continuing seamlessly to infinity. The details of the mathematical procedure employed appear in the Appendix. An important aspect of this transition is that the production of oil each year in the logistical section of the curve will always be greater than it would have been had the parabola continued for the same length of time.

The question then arises as to how the transition point should be chosen. In this note, the point was selected based upon a well-known tendency of decline curves in mature production regions in that the reported Reserves to Production Ratio (R/P) rarely falls below 9. Presumably this constancy reflects the incentive to apply special procedures of one kind or another to continue production as long as possible if only for the recovery of relatively small quantities of oil from so-called ‘stripper’ wells as is frequently the case in the U.S. and increasingly in the Western Canada Sedimentary Basin. In this note however, the R/P ratio of 10 (rather than 9) was selected to determine the transition point for the world as a whole because there will still be reservoirs in production at that time (especially in the Middle East) operating at much higher values of this ratio for many years into the future. A convenient mathematical technique to determine the date of this transition point based upon an R/P ratio of 10 appears in the Appendix.

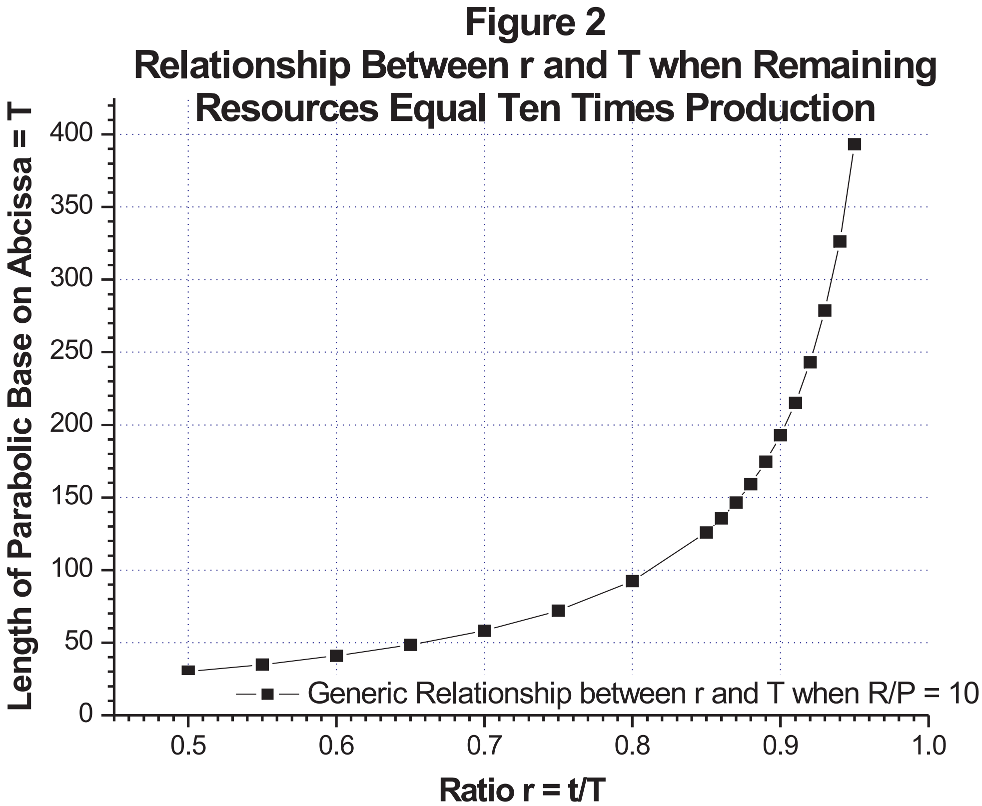
From previous papers, the peak in the world production of conventional oil is expected in the 2015 – 2020 period.[1] Accordingly, four parabolic decline curves appear in Figure 1 starting at peaks in 2010, 2015, 2020 and 2025 to cover a wider range of possibilities for this important date in the world energy system. The timing of the transition point is then determined for each case at the R/P ratio of 10. The slope of the parabolic section is determined from the derivative of the parabolic production curve at the time selected for the transition point. The logistic curve is then drawn starting at this slope at the transition point using the ‘fold-down’ method for symmetric curves as presented in the Appendix.

The transition points between the parabolic and logistic sections of each of the four decline curves so



calculated are indicated with triangles in Figure 1. Diamonds mark the date on each logistic curve when the date at which the area under the logistic curves just equals the area left under the corresponding parabolic curve had they continued past the transition point to the abscissa. This area represents the quantity of oil not produced during the parabolic decline but transferred to the logistic section of the curve. The area under the logistic section later than this point is extra oil not predicted by the parabolic modelling method. Stated another way, this method of modelling the decline of world conventional oil production implies that more oil will be produced over time than the parabolic technique predicts past a certain point. This extra production is restricted to the lower production levels in the so-called 'tails' farther out in time and extends from the diamond markers to infinity. The compound parabolic/logistic curve is a more realistic representation of the situation than the abrupt finish predicted by the parabolic method alone at the abscissa. Although the quantity of oil involved is not large, it may prove important in estimating emissions of carbon dioxide from the world's conventional oil production in the later years of this century. A separate scale is provided to estimate the carbon associated with this oil. It is noteworthy that some authors such as Campbell restrict their study of the decline period to 2075 to avoid the difficulties in estimating this relatively small residual quantity of oil.[3}

There is no reason other modelling curves cannot be joined together to form compound curves in this way including compound logistic/logistic curves if desired.



References

1. J.H. Walsh, *Procedure for the Parabolic Projection of Geological Assessments of Conventional Oil and Gas Resources with Examples*, Revised January 2004 (Web: pages.ca.inter.net/~jhwalsh/wpara1.html). For details of calculation of parabolas with pre-selected peak dates see Appendix 1 to *Note on Remaining World Oil Resources and Per Capita Production Relations Near the Peak,* January 2006. (Web: pages.ca.inter.net/~jhwalsh/wresource-percapita.html)
2. Alex G. Kemp and A.S. Kasim. *Are Decline Rates Really Exponential? – Evidence from the U.K. Continental Shelf*, The Energy Journal, Vol. **26**, No.1, 2005.
3. Colin J. Campbell, *Monthly Newsletter*, World Assessment published in each issue, Association for the Study of Peak Oil and Gas (ASPO). (Web: www.peakoil.ie)

Appendix

Technique for Plotting Compound Parabolic/Logistic Curves

A parabolic production equation is derived for each case of the peak chosen according to the procedure in Reference 1.

This equation takes the form $p = \frac{6Q}{T^2} t \left(1 - \frac{t}{T}\right)$ for the production in GB/year; Q is the total area under the parabola, and T the total length of the intercept with the abscissa from the parabolic start at t = 0.

The derivative dp/dt is a straight line with

$$\frac{dp}{dt} = 6\frac{Q}{T^2} - 12\frac{Q}{T^3} * t$$

The date of the transition time, t, is determined when the ratio of the oil remaining to be produced equals ten times the production as at the right. The slope of the parabola at that time is dp/dt.

The symmetrical logistic curve may be represented as follows:

$$p = \frac{2p_t}{1 + be^{-\alpha t}}$$

where p is the production and t is set at zero at the time of the transition point p_t in the parabolic plot. The constant b =1 for the symmetrical logistic curve case.

The derivative of the logistic curve is then determined:

$$\frac{dp}{dt} = \frac{2p_t \alpha e^{-\alpha t}}{\left(1 + e^{-\alpha t}\right)^2}$$

When t = 0, $\frac{dp}{dt}$ equals $\frac{p_t \alpha}{2}$. This value is set equal to the derivative of the parabolic curve derived above at the same date but with the sign adjusted from negative to positive to account for the reverse direction of the logistic curve before folding. The value of the constant α may then be determined as equal to:

$$\alpha = \frac{2}{p_t} \frac{dp}{dt}$$

It now becomes possible to obtain the values needed to plot an inverse logistic curve that proceeds upwards from the selected transition point to approach the value of $2 * p_t$ at infinity. This curve is then 'folded-down' about the horizontal axis at p_t so as to approach the bottom axis at infinity from the same point p_t at t_p on the parabolic scale which is also 0 on the logistic scale. This is accomplished by subtracting each of the calculated plotting points from $2p_t$.

Selection of the Transition Point

In this note, the transition point p_t for each of the parabolas was selected for the time, t, when the quantity of oil remaining under the parabola was equal to ten times the production so that R/P = 10. Thus 10 times p_t equals the area of the parabola, Q, less the area up to t. To meet this condition, the following equation may be written:

$$10\left[\frac{6Q}{T} r_t \left(1 - r_t\right)\right] = Q - \left[3Qr_t^2 - 2Qr_t^3\right]$$

where $r_t = \frac{t}{T}$. This equation may be simplified to:

$$T = \left(\frac{60r_t - 60r_t^2}{1 - 3r_t^2 + 2r_t^3}\right)$$

A graphical solution for any parabola appears in Figure 2 which relates the value of T to r_t for the case of R/P = 10. Because T is already known from the parabolic equation already derived for each case, it is possible to read off a value for r from the graph. Given this knowledge of r_t, it is then possible to determine the time of the transition point t. The logistic section of the curve may then be determined as above.

The quantity of oil under the logistic curve may be obtained by integrating the logistic equation since the values of α and t are known.

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