Comparison of the Hubbert Linearization (HL) and Staged Parabolic Methods for Determining Peaks in World Oil Production

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Introduction

The Hubbert Linearization and the Staged Parabolic techniques were applied to the same production and resource data so as to provide a direct comparison of the predicted date and value of the peak in the world production of conventional oil estimated by the two methods. The Hubbert Linearization technique (HL) was employed by Professor Kenneth Deffeyes in his influential book *Beyond Oil: The View from Hubbert's Peak* published in 2005[1]. The Staged Parabolic technique was devised by this author to project the results of geological resource assessments as parabolic production plots over time.[2]

The HL procedure converts the annual versus cumulative production parabola into the more useful linear form. This parabola is itself the track of the first derivative of a logistic function. The linear conversion is a great convenience because it allows the easy extrapolation of historical data to predict total ultimate oil production which is often expressed as Ultimate Resource Recovery (URR). In the nomenclature employed here the URR is termed Q as distinct from q, the cumulative production to any given date; these quantities are usually expressed in gigabarrels (GB). Typically the production data, for either the world or for major producing countries such as the U.S.A., is widely scattered above the HL Line in the early years but tends to fall to and lie more closely along the HL line as production matures over time. If there are different episodes in the history of oil production in a given country, such as in the case of Mexico, separate lines may be drawn for each. There is an extensive literature dealing with the application of logistic curves of this type to the study of world oil resources.[3]

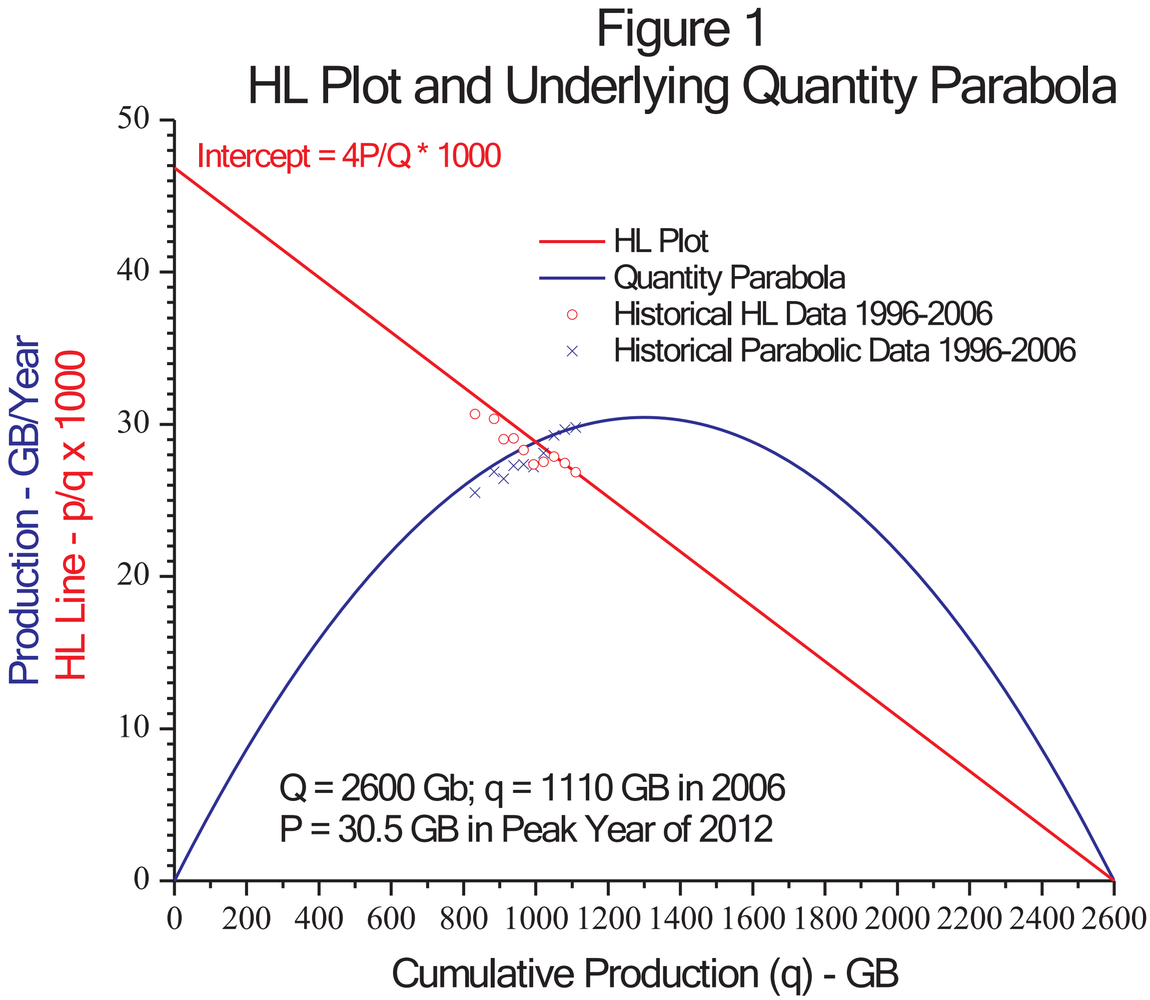
In the case of the Staged Parabola, projected production is plotted against time from the start. The parabolic function was chosen because, in the view of this author, it allows the best representation of the likely conditions near the peak. As a parabola, it will not apply at times distant from the peak but there is less interest in the extreme projection far out in time in any case. No inflection is required in the curve on each side of the peak as compared to, say, some form of the 'bell' curve, and thus there is less constraint preventing it forming a smooth and gently curving peak. Such a function also allows the representation of an 'undulating peak' of the kind expected by some observers such as at Cambridge Energy Research Associates (CERA). There are also advantages in the relatively simple mathematical manipulations allowed by the parabola, including the possibility of using a different but related 'extended' parabola to account for the additional production expected due to the widespread application of the enhanced recovery processes encouraged by higher prices expected after the peak. Various aspects of this technique have been explored in papers by this author posted on his Web Site.

Methodology

Only four basic pieces of data were required for this study. The specific data used in this note may be summarized as:

- Ultimate production Q - 2600 gigabarrels

- Cumulative production to the end of 1996 – 1110 GB. This figure was obtained by extending the data reported by Skrebowski.[4]



- World production in 1996 (25.525 GB) and 2006 (29.807) was taken from the *BP Statistical Review of World Energy* uncorrected for non-conventional oil production.[5]

No elaborate corrections were made to this basic data for non-conventional production because the calculations were intended to be largely illustrative yet dealing with a possible case of interest.

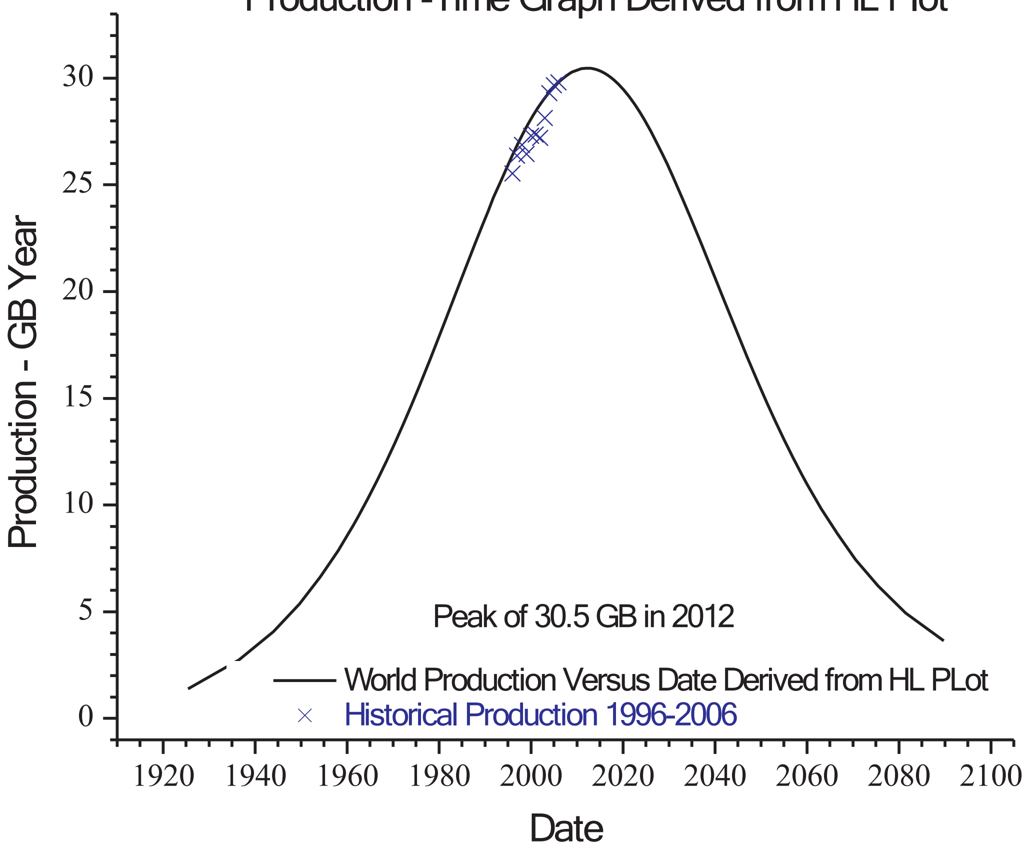
The relationship between the resource parabola (in which production 'p' is plotted versus cumulative production to the same year 'q') and the HL Line was first derived. The basic quadratic formula for a parabola of the type characterized by the anchoring relationship q = 0, p = 0 is:

$$p = aq^2 + bq$$

When p = 0, q also equals Q, with the result that a = -b/Q. For a symmetrical parabola, the peak production designated 'P' occurs when q = Q/2. Substitution of these two equivalences results in the following parabolic equation:

$$p = 4P/Q * q - 4P/Q^2 * q^2$$

Figure 2
Production -Time Graph Derived from HL Plot



This equation may be converted to an HL Plot by dividing by q resulting in the following straight line:

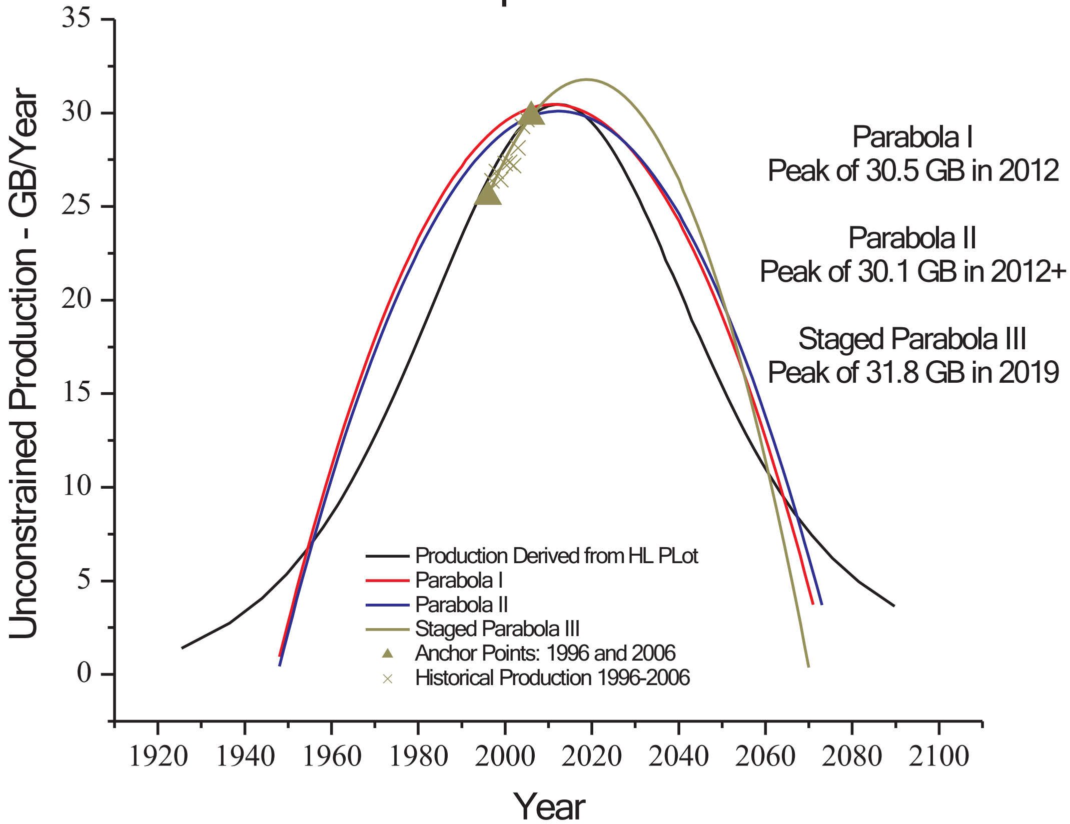
$$p/q = 4P/Q - 4P/Q^2 * q$$

Since p, q and Q are taken as known for 2006 using the values listed above, it is possible to compute the value of the peak, P. The numerical value of the y-axis intercept on the HL Plot is equal to 4P/Q. The x-axis intercept occurs when p = 0 and q = Q, or 2600 GB in this case. It is thus possible to draw the HL straight line by linking these two intercepts.

Both the resource/production parabola and the HL Line derived from it appear in Figure 1 together with historical production data for the decade 1996/2006. It will be noted that the p/q values of the HL Line have been increased by a factor of 1000 to enable plotting on the same graph.

The production-time 'bell' curve may also be derived from the same data. Rather than employ a differential equation, the calculation was conducted here using an incremental slice of 30 GB (which approximates one year's production at the time of the peak) following the procedure reported by de Souza.[6] At the end of the reference year of 2006, the cumulative production was 1110 GB. The time required for the passage of each increment of 30 GB on either side of this reference point was computed from the reciprocal values of the production from the cumulative production parabola. These increments

Figure 3
Three Parabolas Compared to HL-Derived Bell Curve



of time were either added (to extend into the future) or deducted (to determine the past) step-by-step from 2006. The error implicit in using a finite increment of this size instead of a differential function is not considered significant in relation to other uncertainties. The 'bell' curve that results is plotted in Figure 2 including historical production data for the decade 1996/2006.

Three different production/time parabolas were then plotted in Figure 3 in conjunction with this 'bell' curve. In the First Parabola (I), the area under the parabola is the same as that under the bell curve (= Q) with both the value of the peak production and its timing coincident. This may be done simply by employing the following general relationship for parabolic functions:

$$Q = \frac{2}{3} * PT$$

Since Q and P are known, the value of T – the distance between the two intercepts of zero production of a production-time parabola on the x-axis – may be determined. From this value, the starting time of the production/time relationship may also be determined because, with the date of the peak already established, the starting time is T/2 years before. The plot may then be plotted using the following production/time equation which is common to any parabola of the class that results when t = 0 p = 0:

$$p = 6Q/T^2 * t(1 - \frac{t}{T})$$

The Second Parabola (II) was drawn specifying the total cumulative production would be the same as Q (2600 GB) as before but that the cumulative production to the end of 2006 would be set at 1110 GB. This was accomplished by solving the following cumulative production formula using a mathematical program:

$$q = 3Qr^2 - 2Qr^3; r = t/T$$

Once r is determined, T may be determined from the manipulated production/time equation given that the production, p, is known in 2006 as follows:

$$T = 6Q/p * r(1-r)$$

With T and Q now known, the production/time equation may be employed as before.

The Third Parabola (III) is of the staged-type. Here the objective is to draw a parabolic projection from the most recent production data point (2006) in such a way the projected curve embraces the quantity of oil still expected to be produced. This quantity is the difference between the assumed Q and the cumulative production to 2006 thus 2600 – 1110 = 1490 GB. The iterative technique developed to accomplish this result requires an anchor production point usually chosen ten years previously.[2] It will be evident that the Staged Parabolic procedure, unlike the previous cases explored in this note, does not involve the history of oil production except for the period extending back to the anchor point. The severe distortions that occurred at the time of the first oil crisis are thus avoided.

Discussion of Results

The peak in production predicted by the HL Plot of the pre-specified data is 30.5 GB/year occurring in 2012.

In the case of Parabola (I) calculated with the same Q, P and timing as the HL Plot, the shape of the parabola is such that, not surprisingly, much more oil is produced in the years immediately on each side of the peak than when production follows a bell-shaped curve. The line of the parabola crosses over that of the bell curve at a distance from the peak at a time of low production.

In the case of Parabola (II) with the same Q but with cumulative production specified as 1110 GB in 2006, the peak is reduced slightly to 30.1 GB/year with the date shifted somewhat later to 2012.3

A substantial difference occurs in the case of the Staged Parabola (III). The projected peak is increased to 31.8 GB/year which occurs considerably later in 2019.

The main conclusion of this note is that same resource data when projected by the Staged Parabolic technique results in the prediction of a higher and later peak in conventional oil production than the HL method. The production predicted in much later years must thus be less. In a previous note, techniques were explored for compounding logistic curves with the parabolic curves well after the peak to provide a more realistic representation of the far-out decline period.[7]

References

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